Letters to the Editor

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ON THE CLASSICAL RADIATION BY AN ELECTRON IN UNIFORM CIRCULAR MOTION IN THE FORM OF MULTIPOLES

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INTRODUCTION

The problem of classical radiation by electron in uniform circular motion was extensively discussed by Schott (1912). While Schott's interest was to explain atomic structure Schwinger (1912) repeated that analysis of the same problem to find out the maximum energy limit to be obtained by synchro-cyclotron. In this paper attempt has been made to obtain classical radiation by electron in uniform circular motion in the form of multipoles. An electric charge within the medium of dielectric constant ϵ is moving in a circular orbit of radius R with constant angular velocity ω_0. The motion is assumed to be confined in a plane and the angular distributions of the radiation have been obtained after integration of the usual energy term on the surface of a sphere.

Spherical wave solution of Maxwell's Equation :

The Maxwell's equations for a source moving in a circular orbit of radius R and constant angular velocity ω_0 in the medium of dielectric constant ϵ,

$$\nabla^2\phi - \frac{\epsilon}{c^2}\frac{\partial^2\phi}{\partial t^2} = \frac{4\pi}{\epsilon}\rho \quad \ldots \quad (1)$$

$$\nabla^2 A - \frac{\epsilon}{c^2}\frac{\partial^2 A}{\partial t^2} = \frac{4\pi}{c} J \quad \ldots \quad (2)$$

where ϕ, A, ρ, J have their usual meaning. Out of these two equations (1) and

(2) solution of single equation is enough to determine the two quantities ϕ, A, since ϕ, A are related by the Guage

$$A = \frac{\epsilon v}{c}\phi$$

The potentials ϕ, A, and the source density $\rho(x)$, $J(x)$ are transformed in space and time according to the general rule of Fourier transforms.

$$F(x, t) = \frac{1}{(2\pi)^2}\int d^3K \int dw\, F(K, \omega)e^{ik.x - iwt}$$

and the equations (1) and (2) are transformed into

$$\left[K^2 - \frac{\epsilon\omega^2}{c^2}\right]\phi(K, \omega) = \frac{4\pi}{\epsilon}\rho(K, \omega) \qquad \ldots \quad (3)$$

$$\left[K^2 - \frac{\epsilon\omega^2}{c^2}\right]A(K, \omega) = \frac{4\pi}{C}J(K, \omega) \qquad \ldots \quad (4)$$

with reference to a spherical polar co-ordinate system we assign the position of the charged particle at any instant to be at $(R, \omega_0 t, \pi/2)$ while the position of observation point has the co-ordinate (r, ϕ, θ).

From the knowledge of Fourier transforms (Morse *et al*).

$$\rho(K, \omega) = \frac{e}{2\pi}\int e^{-ikR[\sin u \cos(\omega_0 t - v)] - iwt}\,dt \qquad \ldots \quad (5)$$

substituting (5) in (3) and applying the same technique of Fourier transforms, we obtain

$$\phi(\omega) = \frac{2e}{\epsilon(\omega)}\,\frac{1}{(2\pi)^{3/2}}\int\int e^{iK.\times - iKR[\sin u \cos(\omega_0 t - v)] - iwt}\,d^3K\,dt$$

$$= \frac{2e}{\epsilon(\omega)}\,\frac{1}{(2\pi)^{3/2}}\,\Sigma\,\Sigma\,\alpha_{mn}\,\delta(\omega - m\omega_0)e^{im\phi}\,\rho_n^m(\cos\theta)j_n\left(\frac{\sqrt{\epsilon\omega}}{c}R\right)h_n\left(\frac{\sqrt{\epsilon\omega}}{c}r\right) \ldots (6)$$

Multipole Expansion of the Electromagnetic fields

The components of the Electromagnetic fields in (r, ϕ, θ) directions

$$E_r = -\frac{\partial\phi}{\partial r} \quad \because A_r = 0$$

$$E_\varphi = -\frac{1}{C}\frac{\partial A\phi}{\partial t} - \frac{1}{r}\frac{\partial\Phi}{\partial\phi} = -\frac{\epsilon v}{c^2}\frac{\partial\Phi}{\partial t} - \frac{1}{r}\frac{\partial\Phi}{\partial\phi}.$$

$$E_\theta = -\frac{1}{r\sin\phi}\frac{\partial\Phi}{\partial\theta} \quad \because A_\theta = 0$$

$$H_r = -\frac{1}{r\sin\phi}\frac{\partial A\phi}{\partial\theta} = -\frac{\epsilon}{C}\frac{v}{r\sin\phi}\frac{\partial\Phi}{\partial\theta}$$

$$H_\varphi = 0$$

$$H_\theta = \frac{1}{r}\frac{\partial}{\partial r}(rA\phi) = \frac{\epsilon v}{cr}\frac{\partial}{\partial r}(r\Phi)$$

From (6) $\Phi(t)$ is obtained from the rule of Fourier transform

$$\Phi(t) = \frac{1}{\sqrt{2\pi}}\int \Phi(\omega)e^{-i\omega t}\,d\omega \quad \text{as}$$

$$\Phi(t) = \frac{2e}{\epsilon}\frac{1}{(2\pi)^2}\Sigma\Sigma\alpha_{mn}e^{im\varphi}\rho_n{}^m(\cos\theta)e^{-im\omega_0 t}j_n(\sqrt{\epsilon m\beta})h_n\left(\frac{\sqrt{\epsilon m\omega_0 r}}{c}\right)$$

For energy transfer in the radial direction we require only E_φ and H_θ, which are when expanded in multipoles,

$$E_\varphi n, m = +im\left[\frac{\epsilon v}{c^2}\omega_0 - \frac{1}{r}\right]\phi_{nm}(t)$$

$$\Sigma\Sigma\ E_{\varphi nm} = \frac{2eim}{\epsilon(2\pi)^2}\left[\frac{\epsilon v}{c^2}\omega_0 - \frac{1}{r}\right]\ \Sigma\Sigma\alpha_{mn}e^{-im\omega_0 t}\times e^{im\varphi}P_n{}^m\ (\cos\theta)$$

$$\times j_n(\sqrt{\epsilon m\beta})\ i^{-(n+1)}\ \frac{e^{\frac{i\sqrt{\epsilon m\omega_0}}{c}r}}{\sqrt{\epsilon m\omega_0}\ r} \qquad \dots (7)$$

$H_{\theta n,m} = \dfrac{\epsilon v}{c}\dfrac{\partial}{\partial r}\phi_{nm}(t)$ neglecting other terms.

$$\Sigma\Sigma H_{\theta nm} = \frac{\epsilon v}{c}\frac{\partial}{\partial r}\Sigma\Sigma\phi_{nm}(t)\alpha_{mn} \qquad \dots (8)$$

Angular distribution of multipole radiation

The electromagnetic fields as obtained from (7) and (8) when terms upto $\left(\frac{1}{r}\right)$ are retained

$$E_\varphi \to \frac{2e}{\sqrt{\epsilon}}\cdot\frac{\beta}{(2\pi)^2}\ \Sigma\Sigma\ \alpha_{mn}i^{-n}\ \frac{e^{\frac{i\sqrt{\epsilon m\omega_0}}{c}r - im\omega_0 t}}{-}\ j_n(\sqrt{\epsilon m\beta})e^{im\varphi}\ P_n{}^m\ (\cos\theta)$$

$$H_\theta \to 2e\cdot\frac{\beta}{(2\pi)^2}\ \Sigma\Sigma\alpha_{mn}i^{-n}\ \frac{e^{\frac{i\sqrt{\epsilon m\omega_0}}{c}r - im\omega_0 t}}{r}\ j_n(\sqrt{\epsilon m\beta})e^{im\varphi}\ P_n{}^m(\cos\theta)$$

The time averaged power radiated per unit solid angle is

$$\frac{dp}{d\Omega} = \frac{e^2\beta^2}{\sqrt{\epsilon}(\pi)^2} j_n^2(\sqrt{\epsilon}m\beta) \, |X_{m,n}(\theta, \phi)|^2$$

The table lists some of the simpler angular distributions.

$X_{m,n}(\theta, \phi)^2$

	m	
	± 1	± 2
1 Dipole	$\frac{3}{16\pi}(1+\cos^2\theta)$	
2 Quadrupole	$\frac{5}{16}(1-3\cos^2\theta +4\cos^4\theta)$	$\frac{5}{16\pi}(1-\cos^4\theta)$

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